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Development of software for high-precision LLR data analysis

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Abstract. *In order to determine the lunar orbital/rotational motion and tidal deformation using lunar laser ranging (LLR) observation data, analysis software is being developed. As the first step of the study, we construct an LLR observation model, combining the newest physical models.*
The observation model consists of the lunar orbit and libration obtained from DE430 (provided by NASA JPL), and the other physical models compatible with IERS Conventions (2010) such as Earth orientation, solid Earth/Moon tides, and some factors affecting propagation delay. For the purpose of calculating these components precisely, we use the modules of the geodetic data analysis software "c5++" (Otsubo et al., 2011). LLR observation data are provided as normal points obtained at Apache Point, Grasse, Matera and McDonald. In this calculation, there are 3372 normal points distributed from June 1996 to August 2012. Comparing the observed and calculated one-way range, the mean and the standard deviation of the residuals are about 5.7 cm and 4.8 cm respectively.

Introduction

This study is composed of three steps; the first is creating the LLR observation model, the next is numerically integrating the lunar orbit and rotation simultaneously, and the last is solving the inverse problem of lunar motion and determining the parameters. The first step has almost finished, and this paper presents the details of that modeling. We are currently working on the next step, the integration of the lunar motion.

The components of the observation model

The computation of the range between the ground observatories and the lunar retroreflector arrays depends on the geocentric locations of the observatories, selenocentric positions of the reflectors, the lunar orbit around the Earth, and the lunar libration with respect to the Earth. Table 1 lists the components in our model, and the orders of magnitudes of these components are given in one-way laser range.

In order to obtain the lunar orbit around the Earth and the libration Euler angles (θ, φ, ψ), we use the JPL lunar and planetary ephemeris DE430 (Williams et al., 2013). Lunar retroreflector coordinates are from Williams et al., (2013) which were determined during the calculation leading to DE430. Transformation from the selenocentric Principal Axis Frame (PA) to the International Celestial Reference Frame (ICRF) is presented as following equation. *R* represents the rotational matrix around the principal axis.

$$[ICRF] = R_3(-\varphi)R_1(-\theta)R_3(-\psi)[PA]$$

Coordinates and velocities of three ranging stations except for the Apache Point are from ITRF2008. For the Apache Point, which is not included in ITRF2008, we used the coordinate and estimated velocity provided by Prof. Jürgen Müller, Leibniz Universität Hannover.
Station eccentricities need to be added to these ITRF coordinates.

Solid Earth tides are calculated by using the modules of the analysis software “c5++” according to the IERS Conventions (2010) model. For the tidal deformation of the lunar retroreflector arrays, we used the model in Murphy et al. (2013). The tidal Love numbers h_2 and l_2 of the Moon are from Williams et al., (2013).

$$h_2 = 0.0476, \quad l_2 = 0.0107$$

Table 1. The components of our model

<table>
<tr><th colspan="2">Components</th><th>Effects on one-way range</th><th>References</th></tr>
<tr><td colspan="2">Lunar orbit around the Earth</td><td></td><td>DE430</td></tr>
<tr><td colspan="2">Lunar libration</td><td></td><td>DE430</td></tr>
<tr><td colspan="2">Retroreflector coordinates</td><td></td><td>Williams et al. (2013)</td></tr>
<tr><td colspan="2">station coordinates and velocities</td><td></td><td>ITRF2008,
personal communication with Prof. Müller</td></tr>
<tr><td colspan="2">Earth rotation and orientation</td><td></td><td rowspan="5">IERS Conv. (2010)</td></tr>
<tr><td rowspan="4"></td><td>Precession</td><td>~ 100 m</td></tr>
<tr><td>UT1 – UTC</td><td>~ 100 m</td></tr>
<tr><td>Nutation</td><td>~ 10 m</td></tr>
<tr><td>Polar motion</td><td>~ 10 m</td></tr>
<tr><td colspan="2">Aberration of light</td><td>~ 1 m</td><td></td></tr>
<tr><td colspan="2">Tropospheric delay</td><td>~ 1 m</td><td>IERS Conv. (2010)</td></tr>
<tr><td colspan="2">Relativistic propagation delay</td><td>~ 1 m</td><td>IERS Conv. (2010)</td></tr>
<tr><td colspan="2">Lorentz transform between TDB and TT</td><td></td><td></td></tr>
<tr><td rowspan="2"></td><td>Time component</td><td>~ 1 m</td><td>Kovalevsky et al. (1989)</td></tr>
<tr><td>Spacial components</td><td>~ 1 m</td><td>IERS Conv. (2010)</td></tr>
<tr><td colspan="2">Solid Earth tides</td><td>~ 0.1 m</td><td>IERS Conv. (2010)</td></tr>
<tr><td colspan="2">Solid Moon tides</td><td>~ 0.1 m</td><td>Murphy et al. (2011)</td></tr>
</table>

Earth orientation is also calculated by the modules of analysis software “c5++”, with using the long-term Earth orientation parameters EOP08C04 that is distributed by the International Earth Rotation and Reference Systems Service (IERS).
Aberration of laser light is caused by the Earth rotation during the laser round-trip. To compensate it, we solve the equation of light time iteratively by using Newton method.

In addition to the effects caused by the motions of the stations and the retroreflector arrays within the celestial reference frame, various factors such as tropospheric and relativistic delay affect the laser flight time.
Tropospheric delay is corrected by using the Mendes and Pavlis model in IERS Conventions (2010).

The relativistic propagation delay refers the distortion of the laser path in a curved spacetime. The largest gravity source is the Sun, which has the effect of ~ 7 - 8 cm delay on one-way range. We consider the Sun, the Earth (~ 30 – 40 cm) and the Moon (~ 0.1 mm) as the gravity sources.

Treatment of the time scales is also important. The time scale that the ephemeris DE430 is based on is the Barycentric Dynamical Time (TDB). On the other hand, round-trip time is measured by using ground-based atomic clocks that tick in Terrestrial Time (TT). To compensate this difference, spacetime transformation needs to be considered.
Lorentz transformation is the relativistic linear transformation for four-dimensional spacetime. The approximate method for the special component of this transformation is presented in IERS Conventions (2010). For the correction of the time component, we used the numerical approximate model in Kovalevsky et al. (1989).

Results of the modeling

Figures 1 and 2 show the residuals between the one-way observed and calculated ranges. The mean value is 5.7 cm, and the standard deviation is 4.8 cm (with 11 % data rejected). We note that these results are with no parameter estimation.
There are 3372 normal points obtained at Apache Point, Grasse, Matera and McDonald, distributed from June 1996 to December 2012.
The remaining factors are ocean tide loading, atmospheric loading, and target signatures. These have the effects of less than a few cm on one-way range.

Figure 1. One-way residuals (observed – calculated) by stations

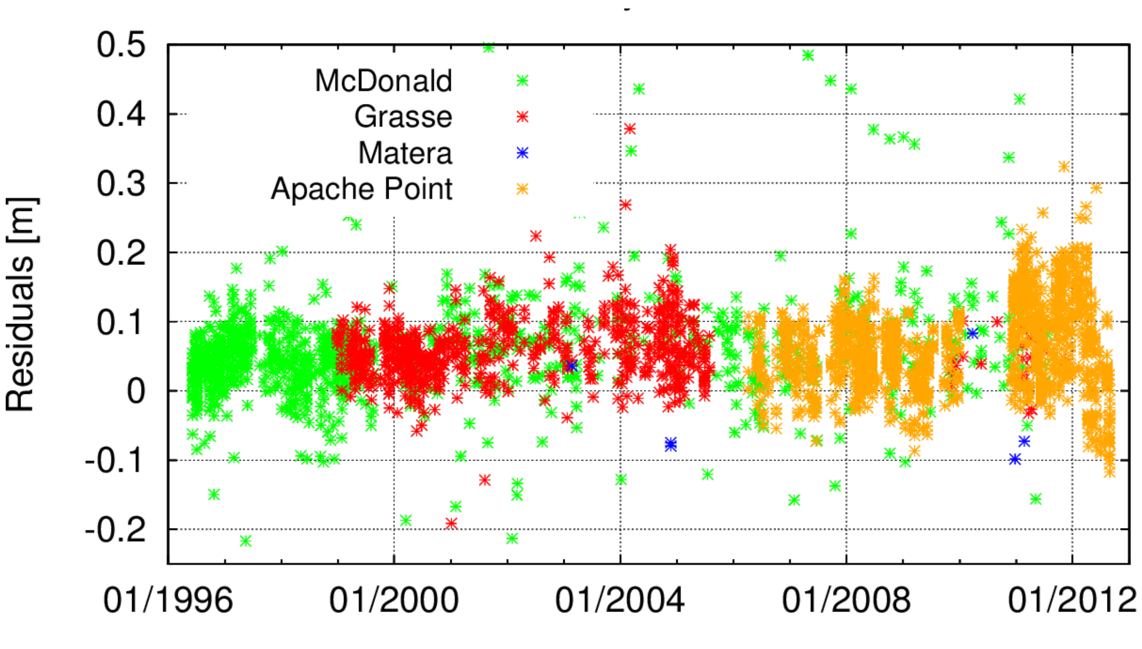
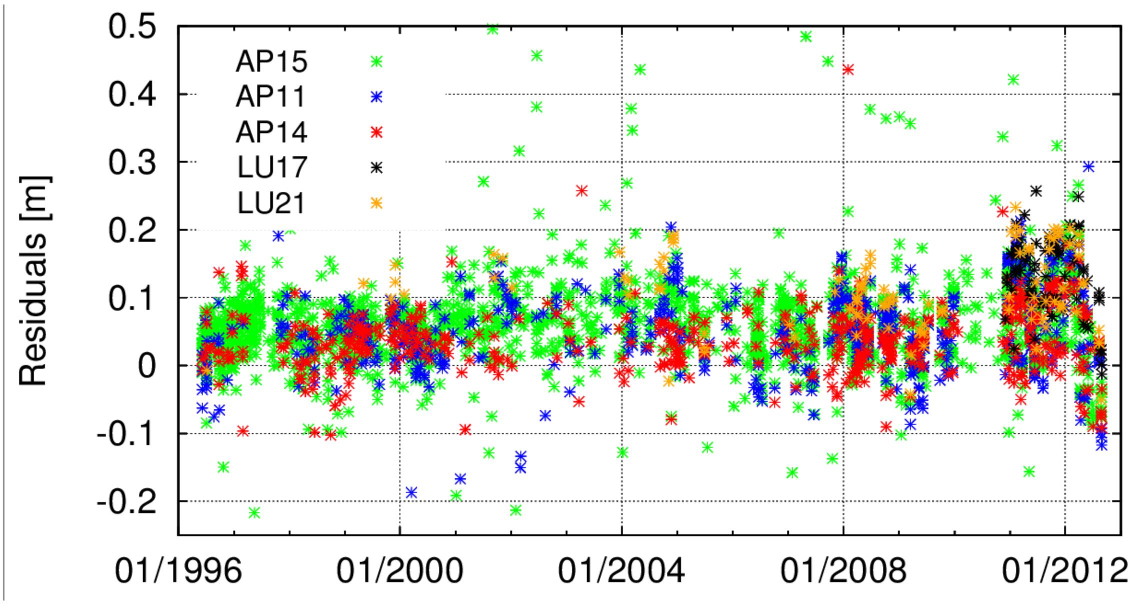


Figure 2. One-way residuals (observed – calculated) by retroreflector arrays



Acknowledgements

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Data source of LLR Normal Points obtained at the three observatories except for Apache Point is the Crustal Dynamics Data Information System (CDDIS), and data obtained at Apache Point are available at the observatory’s web page.

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